ECS315 2014/1 Part VII Dr.Prapun

13 Three Types of Random Variables

13.1. Review: You may recall[51] the following properties for cdf of discrete random variables. These properties hold for any kind of random variables.

(a) The cdf is defined as $F_X(x) = P[X \leq x]$. This is valid for any type of random variables.

(b) Moreover, the cdf for any kind of random variable must satisfies three properties which we have discussed earlier:

CDF1 F_X is non-decreasing

CDF2 F_X is right-continuous

CDF3 $\lim\limits_{x\to-\infty} F_X(x) = 0$ and $\lim\limits_{x\to\infty} F_X(x) = 1$.

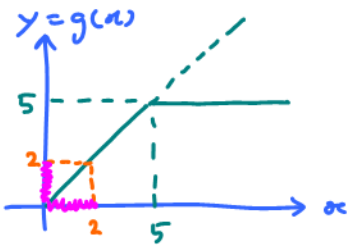
(c) $P[X = x] = F_X(x) - F_X(x^-)$ = the jump or saltus in F at x.

Theorem 13.2. If you find a function F that satisfies CDF1, CDF2, and CDF3 above, then F is a cdf of some random variable.

[51] If you don't know these properties by now, you should review them as soon as possible.

Example 13.3. Consider an input X to a device whose output Y will be the same as the input if the input level does not exceed 5. For input level that exceeds 5, the output will be saturated at 5. Suppose $X \sim \mathcal{U}(0,6)$. Find $F_Y(y)$.

$X \to \boxed{g(\cdot)} \to Y$



① $F_Y(y) = P[Y \le y]$ $\quad y < 0$

$= 0$

② $F_Y(y) = 1$ $\quad y \ge 5$

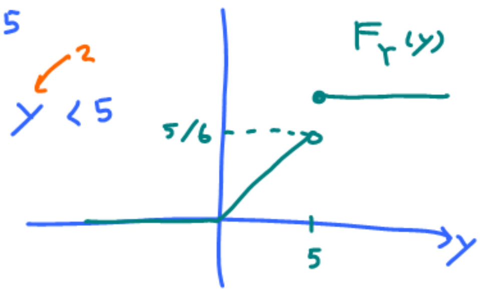
③ $F_Y(2) = P[Y \le 2]$ $\quad 0 \le y < 5$

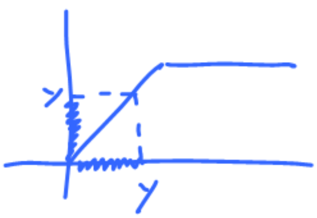
$= P[X \le 2]$

$= \frac{1}{3}$

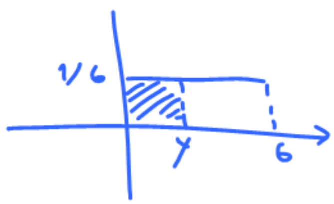
$F_Y(y) = P[X \le y]$

$= \frac{y}{6}$





$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \ge 0 \end{cases}$



$\frac{d}{dt} u(t) = \delta(t)$

$\int_{-\infty}^{t} \delta(x)\,dx = u(t)$

$f_Y(y) = \begin{cases} 1/6, & 0 < y < 5 \\ 0, & \text{otherwise} \end{cases} + \frac{1}{6}\delta(y-5)$

13.4. We can categorize random variables into three types according to its cdf:

(a) If $F_X(x)$ is piecewise flat with discontinuous jumps, then X is **discrete**.

(b) If $F_X(x)$ is a continuous function, then X is **continuous**.

(c) If $F_X(x)$ is a piecewise continuous function with discontinuities, then X is **mixed**.

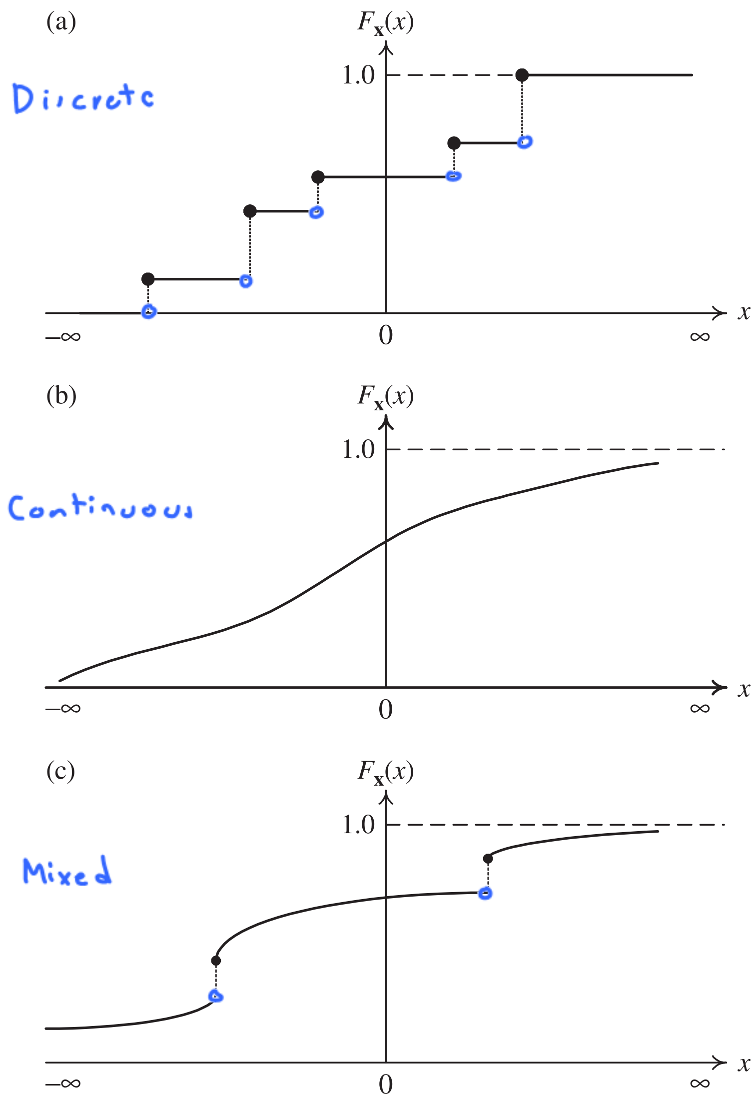
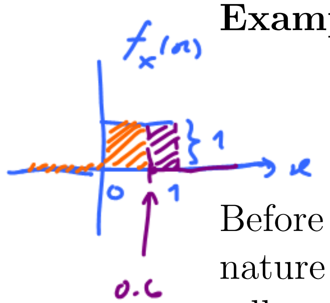


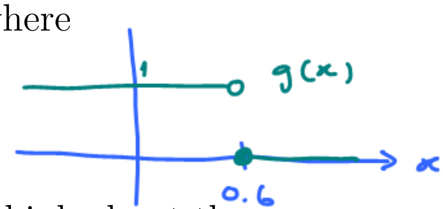
Figure 27: Typical cdfs: (a) a discrete random variable, (b) a continuous random variable, and (c) a mixed random variable [16, Fig. 3.2].

We have seen in Example 13.3 that some function can turn a continuous random variable into a mixed random variable. Next, we will work on an example where a continuous random variable is turned into a discrete random variable.

Example 13.5. Let $X \sim \mathcal{U}(0,1)$ and $Y = g(X)$ where



$$g(x) = \begin{cases} 1, & x < 0.6 \\ 0, & x \geq 0.6. \end{cases}$$



Before going deeply into the math, it is helpful to think about the nature of the derived random variable Y. The definition of $g(x)$ tells us that Y has only two possible values, $Y = 0$ and $Y = 1$. Thus, Y is a discrete random variable.

$P[X \geq 0.6]$

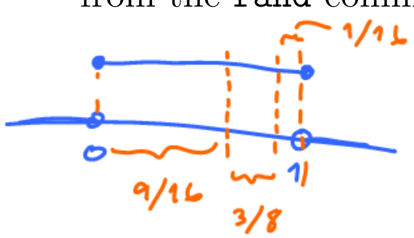
$$p_Y(y) = \begin{cases} 0.4 & y = 0, \\ 0.6 & y = 1, \\ 0, & \text{otherwise} \end{cases} \qquad Y \sim \text{Bernoulli}(0.6)$$

Example 13.6. In `MATLAB`, we have the `rand` command to generate $\mathcal{U}(0,1)$. If we want to generate a Bernoulli random variable with success probability p, what can we do?

$$x = (\text{rand} < p)$$

$$p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

Exercise 13.7. In `MATLAB`, how can we generate $X \sim \text{binomial}(2, 1/4)$ from the `rand` command?



$$p_X(x) = \begin{cases} 9/16, & x = 0, \\ 3/8, & x = 1, \\ 1/16, & x = 2, \\ 0, & \text{otherwise} \end{cases}$$

$$\mathbb{E}X = n \times p = 2 \times \frac{1}{4} = \frac{1}{2}$$

$$\text{Var}\, X = n \times p \times (1-p) = 2 \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{8}$$

$$X = \begin{cases} 0, & \text{if } 0 \leq \text{rand} \leq \frac{9}{16}, \\ 1, & \text{if } \frac{9}{16} < \text{rand} \leq \frac{15}{16}, \\ 2, & \text{if } \frac{15}{16} < \text{rand} \leq 1, \end{cases}$$